Lensing of gravitational waves in self-lensing systems

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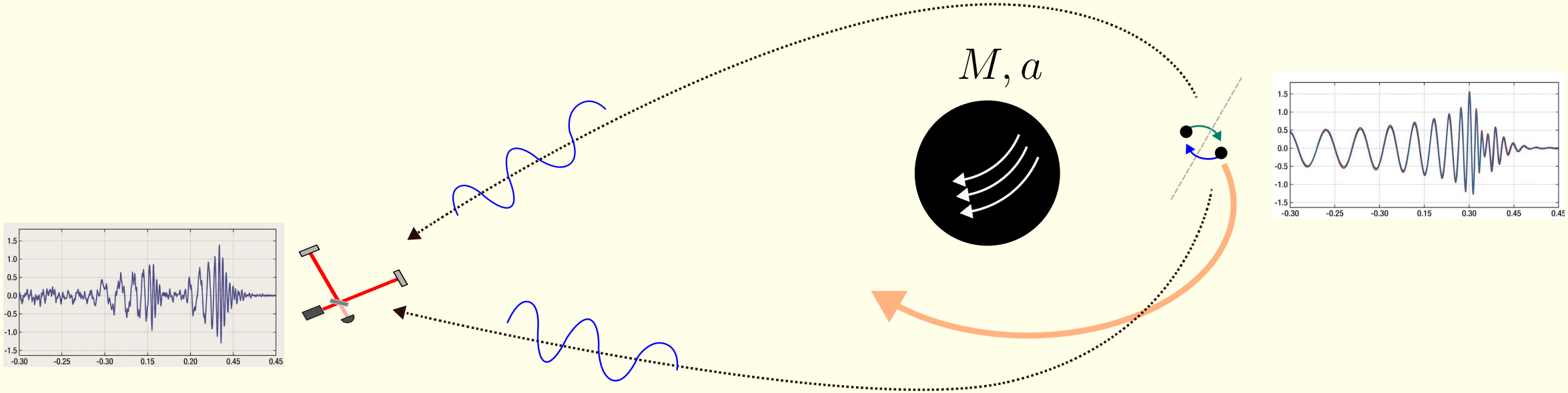
WEAVE Unisono project NCN 2024/06/Y/ST2/00190 *„Lensing of electromagnetic and gravitational waves”*

Lensing and gravitational wave propagation

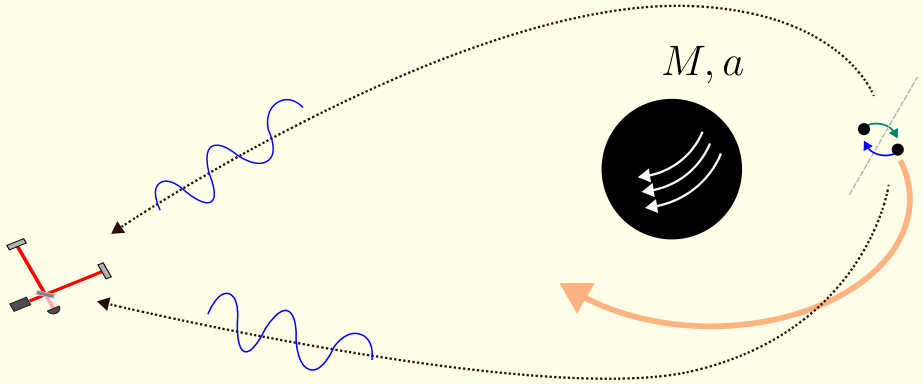
- Gravitational waves undergo lensing just like electromagnetic waves
- If the wavelength is small in comparison with the curvature scale/lens size ($\epsilon = \frac{\lambda}{D} \ll 1$) we may use geometric optics approximation
  - waves follow rays (null geodesics), we can use ray tracing
  - propagation independent of polarization
- If not then the problem is much more complex ($O(\epsilon)$ effects present)
  - **diffraction effects**: inference between rays
  - **spin-Hall effect**: different circular polarisations travel differently through spacetime
- Does this matter? Possibly **yes**, in hierarchical triples

Hierarchical triple merger



Question: unlensed signal near the BBH $\longrightarrow$ lensed signal at the observation point

Hierarchical triple mergers - physics



- motion effect
  - Doppler effect
  - beaming effect
  - time-dependent time delays
- multiple imaging
- maginification, caustics
- viewing angle variations [MK, Kulejewski 2026]
- precession of the BBH system
- spin-Hall effect [Andersson, Oancea 2023]

Hierarchical triple mergers

Lensing in these systems considered before:

- Ubach 2025 geometrical optics
- Savastano *et al* 2024 - Sgr A*, continuous waves
- Kubota *et al* 2024, Joao Santos *et al* 2025, BH perturbation theory

Can such triple systems exist?

- formation channels include:
  - Migration traps in AGN disks [Bellovary *et al* 2016, McKernan *et al* 2012], ...
  - Disk capture of compact objects in galactic nuclei clusters [Wang *et al* 2023], ...
  - von Zeipel–Lidov–Kozai oscillations of BBH [Antonini, Perets 2012]
  - ...

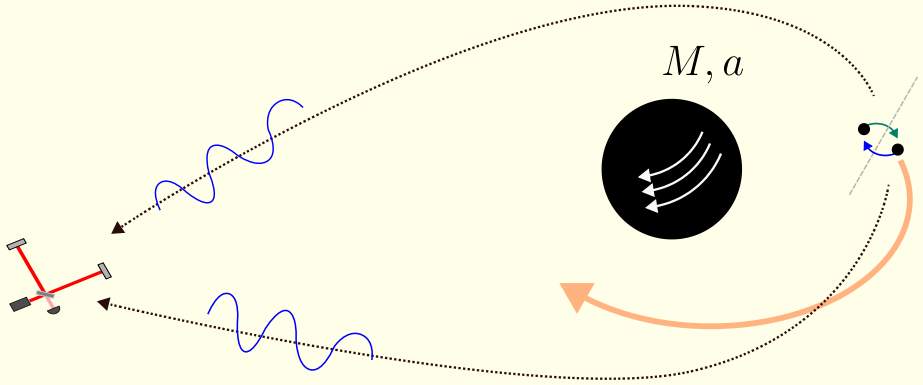
What can we learn from them?

Questions:

- can we identify this type of hierarchical mergers or continuous waves?
- what kind of information can we extract from the signal?
- can we measure the post-geometrical optics effects (spin-Hall)?

Fundamental problem

- unlensed signal near the BBH $\longrightarrow$ lensed signal at the observation point



Methodology

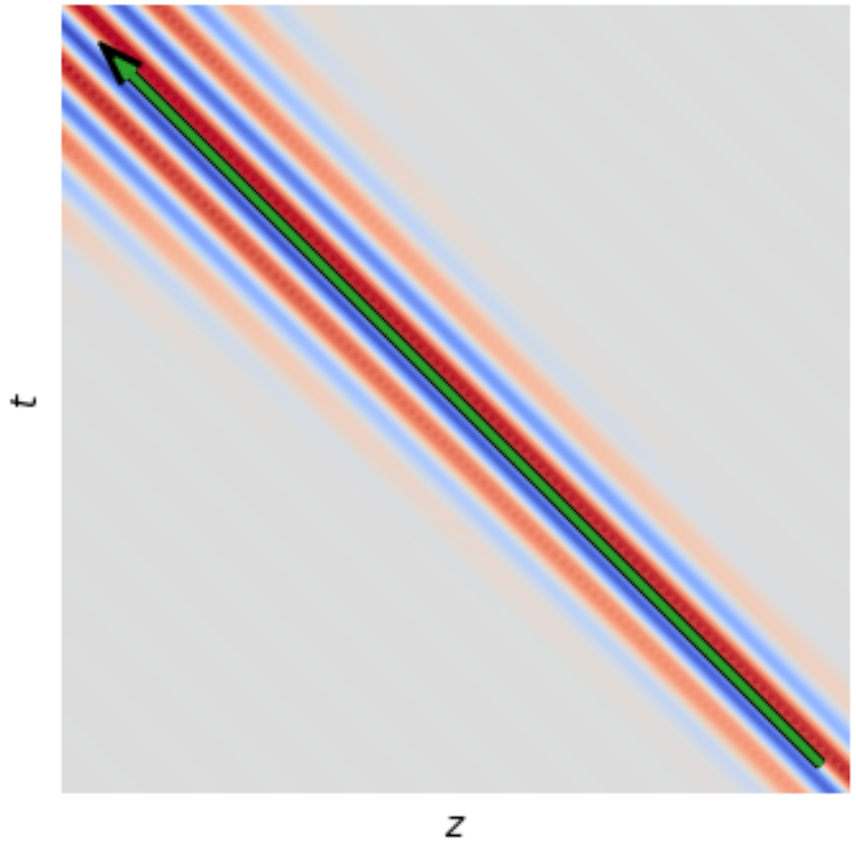
- Solve the wave equation on a Kerr background directly (with a moving source) [Kubota *et al* 2024], [Santos *et al* 2025]
  - computationally very demanding
- Standard ray tracing
  - assumes geometric optics approximation
    $\Longrightarrow$ we lose some important effects

Solution

- Smarter ray tracing: **Gaussian beam approximation** (GBA)

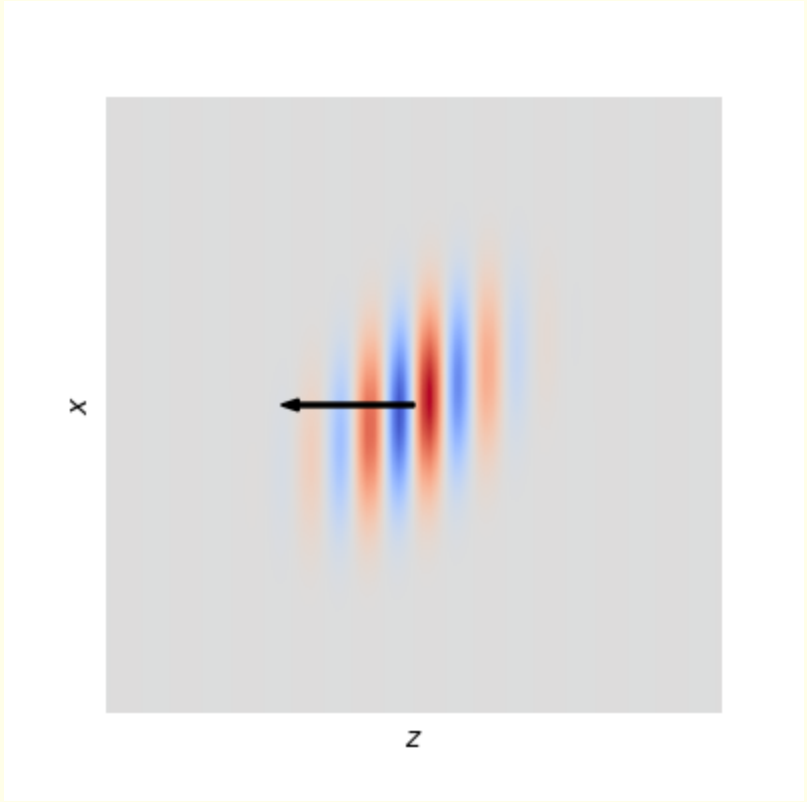
Gaussian beam approximation

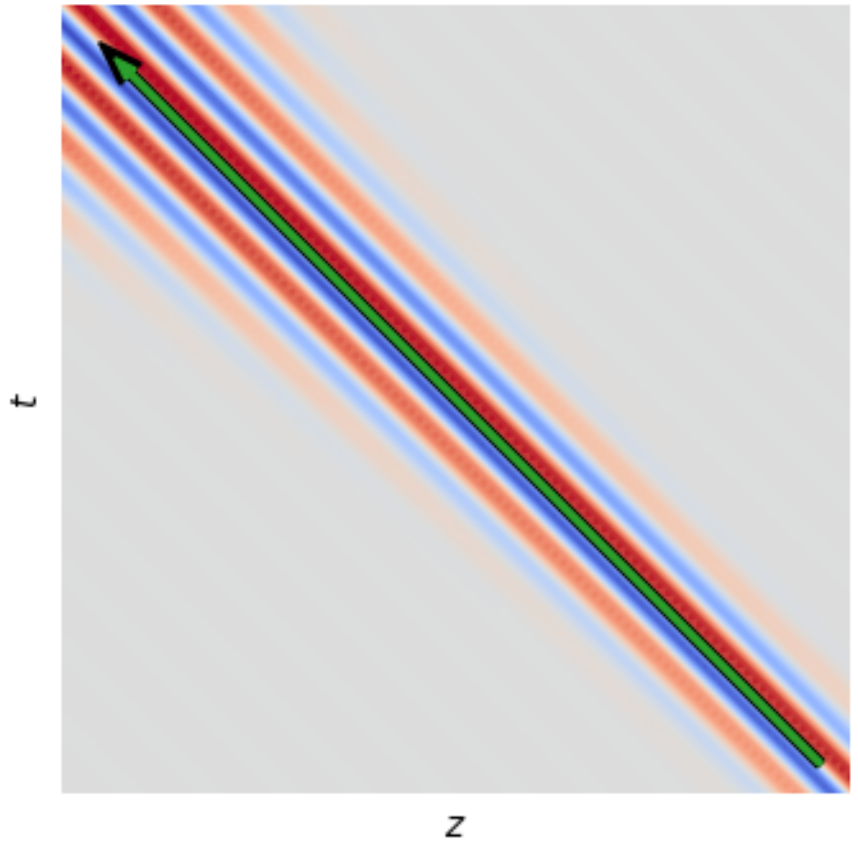
- Similar to the standard geometric optics, but we assume a spatially localized wave packet rather than a plane wave
    - plane and spherical waves can be decomposed into Gaussian packets
- Requires solving the null geodesic equation, and then ODE's along this geodesic



Gaussian beam approximation

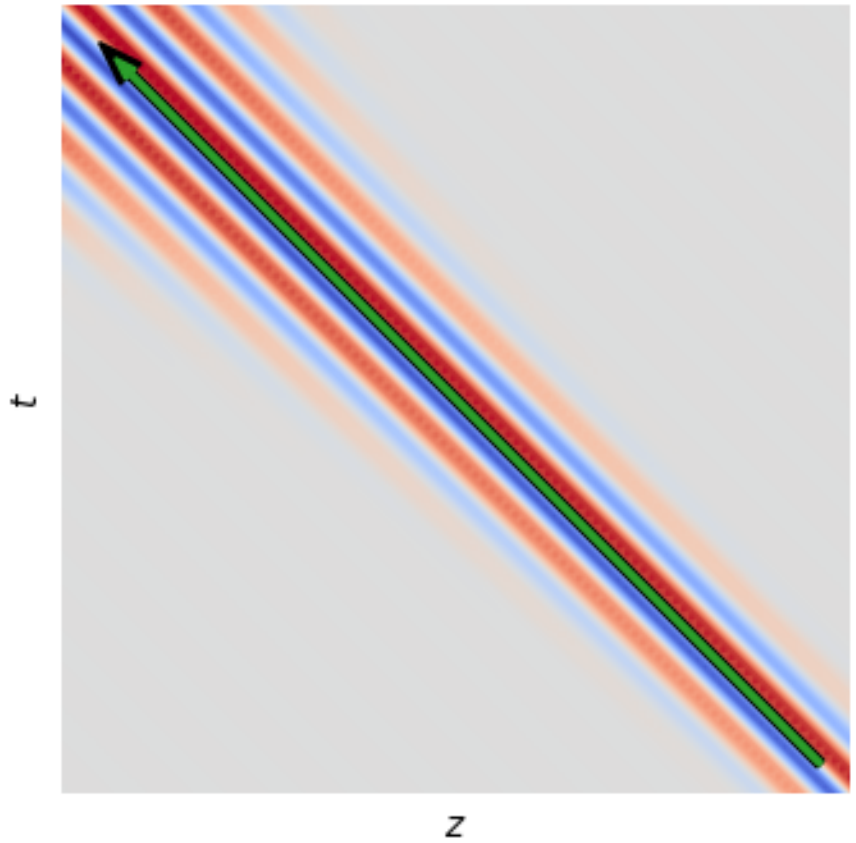
snapshot of the wave packet:



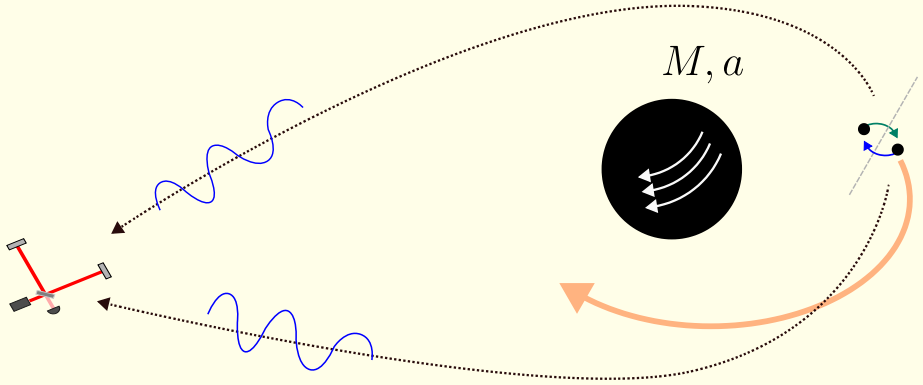


Gaussian beam approximation

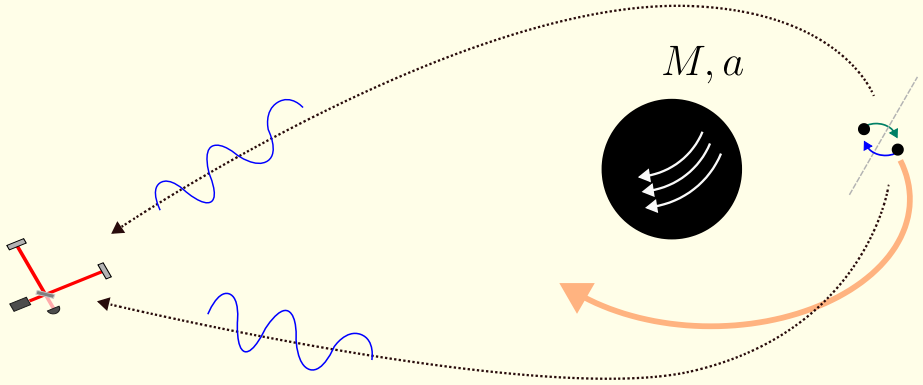
- Well-behaved when passing through caustics, includes automatically the geometric phase
- Works as a (well-behaved) asymptotic expansion in a small parameter
  - $\epsilon = \frac{\lambda}{D}$, D - curvature scale
- Easy to include higher order terms in the expansion (spin-Hall)
- Large system of ODEs (7 tensors, 135 components), but numerically well-behaved



The code



- `GBLensing`, not public yet
- Written in Julia
- Authors: Haider Khan, Chris Harvey-Hawes, MK, Marius Oancea
- based partially on earlier work of Richard Stiskalek `GSHE`
- implements the GBA up to the subleading order in a Kerr metric
- static point source only (at the moment)
- computes the signal in a detector positioned anywhere using ray-tracing



Future directions

- include the motion effect of the point source
- search for the effects of motion in the distorted signal
- search for the spin-Hall effect in the distorted signal
- study the feasibility of observations

Thank you!